

**LESSON 26 – THE DOPPLER EFFECT**

*In previous lessons, we've hinted at the huge effects of target and shooter velocity on radar returns. In this lesson we'll see just what those effects are.*

**Reading:**

Stimson **Ch. 15**

**Problems/Questions:**

Work on Problem Set 4

**Objectives:**

Understand what the Doppler effect is and what causes it.

Understand how a Doppler shift occurs for a moving radar.

Understand how phasors can be used to represent a Doppler shift in EM waves.

Be able to calculate the Doppler shift of a radar return.

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Last Time: Chirp/Pulse compression  
FM Ranging

Today: Doppler shift  
wave explanation  
phasor explanation

Equations:  $\Delta f = \frac{-2\dot{R}}{\lambda}$

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Show F-4 intercept tape

discuss ground clutter, all facets of the display

Show F-15 intercept tape

the advantage is obvious

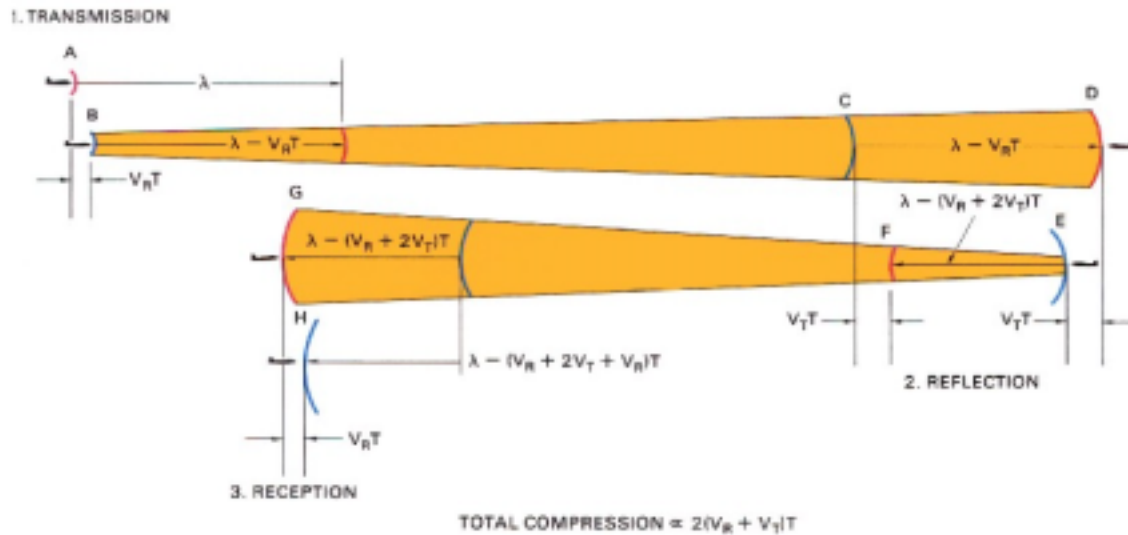
So far we've looked at range. Now we turn to range rate, or velocity.

We've previously alluded to the fact that a moving source of waves causes the waves to be compressed in the direction of the source's motion. This is called the Doppler Effect.

Show "Doppler stationary.avi" and "Doppler moving.avi"

Ground-based radar only has one velocity to deal with – the velocity of the target. Airborne radar has two – the target's velocity and the radar's velocity.

Show overhead of Stimson figure 15.3, p. 191.



This says that any wave is compressed by  $\Delta\lambda = 2(\dot{R})T$ .

What does this tell us about the frequency shift? Don't worry about knowing this derivation, but we'll only use  $c = \lambda f$ ,  $T = 1/f$ , and the concept of derivatives to get a relatively simple result for the frequency shift.

$$c = \lambda f \Rightarrow f = c/\lambda$$

$$df = d(c/\lambda) = c d(1/\lambda) = c d(\lambda^{-1}) = -c (\lambda^{-2}) d\lambda = \boxed{-c d\lambda/\lambda^2 = df} \quad (\text{equation 1})$$

We already know an equation for  $d\lambda$ , that is,  $\Delta\lambda = 2(\dot{R})T$

$$\text{Now, } T = 1/f, \text{ so } \Delta\lambda = \frac{2(\dot{R})}{f} = \frac{2(\dot{R})}{\left(\frac{c}{\lambda}\right)} = \boxed{\frac{2(\dot{R})\lambda}{c} = \Delta\lambda} \quad (\text{equation 2})$$

Plugging equation 2 into equation 1 after converting from differentials, we get

$$\Delta f = \frac{-c}{\lambda^2} (\Delta\lambda) = \frac{-c}{\lambda^2} \left( \frac{2\dot{R}\lambda}{c} \right) \text{ so } \boxed{\Delta f = \frac{-2\dot{R}}{\lambda}} \quad \text{This is the Doppler frequency shift}$$

in terms of closure rate. It's the same equation that Stimson gets.

A quick look at phasors and Doppler – quick but VERY important for our follow-on study of digital radar.

The physical concept of Doppler is best understood by looking at wavelength compression, as we have just done.

The way Doppler is used by digital filters is best seen with phasors, and by looking at a Doppler shift as a continuous shift in phase.

Our purpose: To show that a rotating phasor, i.e. a continuous phase shift, equates to a Doppler shift.

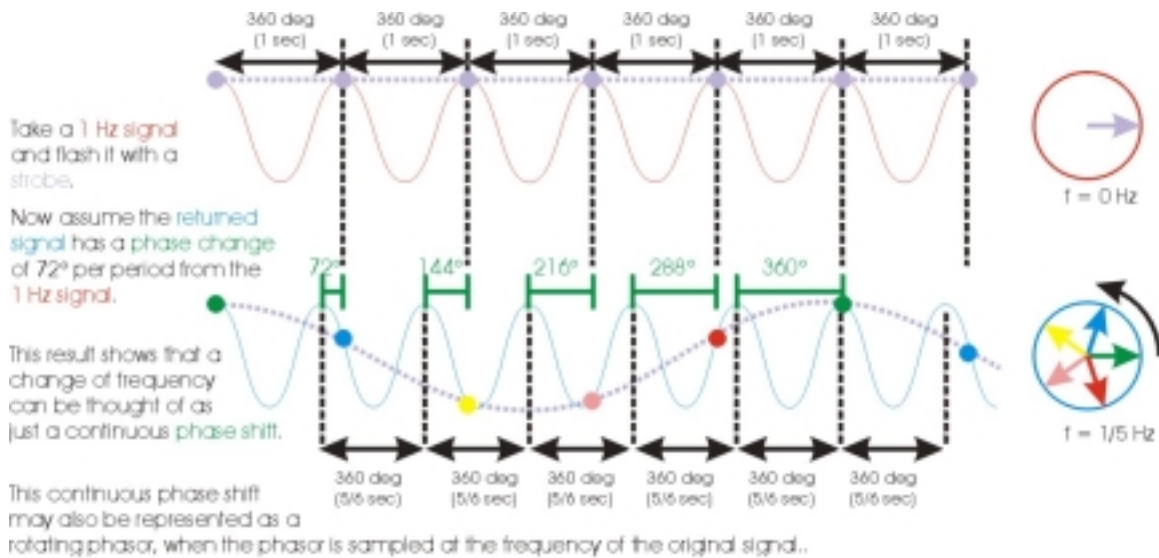
**Show the bicycle wheel with the phasor arrow on it.**

If the wheel spins at 1 Hz, how can we make it appear to stand still? We can strobe it at 1 Hz. We TUNE the strobe to the frequency of the wheel.

If we strobe the wheel at the same rate but now spin the wheel at a different rate, say, 1.2 Hz, what does the phasor do? It rotates at the difference frequency,  $0.2 \text{ Hz} = 72 \text{ deg/sec}$ .

On a phasor, what represents phase? The angle with respect to the positive x-axis. So in this case the rotating phasor is out of phase with the strobe by 72 deg per rotation...

How does this continuous phase shift shown by the phasor rotation relate to the Doppler shift from our wavelength compression example? Let's look at our 1 Hz signal for 5 seconds...



So by having our phasor rotate at 72 deg/sec, we end up with a wave with a period of 5/6 sec, or a frequency of 1.2 Hz, as advertised. Phasors rotating with respect to one another can illustrate a Doppler shift. Anything that causes wavelength compression/expansion will cause a phasor to appear to rotate with the difference frequency if the phasor is sampled at the frequency of the original signal.

How much is a typical Doppler shift for an intercept? More rule of thumb numbers: Typical radar:  $f = 10 \text{ GHz}$

Typical intercept closure:  $\dot{R} = 1000 \text{ kts} = 515 \text{ m/sec}$

Substitute this into the Doppler equation, and we get

$$\Delta f = \frac{-2\dot{R}}{\lambda} = -2(515 \text{ m/sec})/0.03 \text{ m} = 34,333 \text{ Hz, or } 34 \text{ kHz.}$$

The rule of thumb to remember is that *Doppler shifts seldom exceed 100 kHz.*

Review rules of thumb:

$$f_{\text{radar}} = 10 \text{ GHz}$$

$$f_{\text{Doppler}} < 100 \text{ kHz}$$

$$\lambda_{\text{radar}} = 3 \text{ cm}$$

$$D_{\text{radar}} = 1 \text{ m}$$

$$V_{\text{intercept}} = 1000 \text{ kts}$$

A closure rate will cause the phasor to rotate, but which way? We just showed that a wavelength compression caused the phasor to rotate with a frequency of  $+f_{\text{difference}}$  (the phase of the compressed wave led the phase of the transmitted wave by 72deg/sec), which gave us CCW rotation of the phasor. What causes compression – opening or closing velocities? Closing, so

Closure => CCW rotation of the received phasor

Opening => CW rotation of the received phasor

**Show double vibrator/strobe demo**

Why won't this tell us whether we have opening or closing velocities? It's only one dimensional. To get CCW or CW, we need two dimensions, or two equations => motivation for in-phase/quadrature (I/Q) analysis.

Draw the res-cell tactics again and show why they worked (Doppler masking, beamwidth considerations)